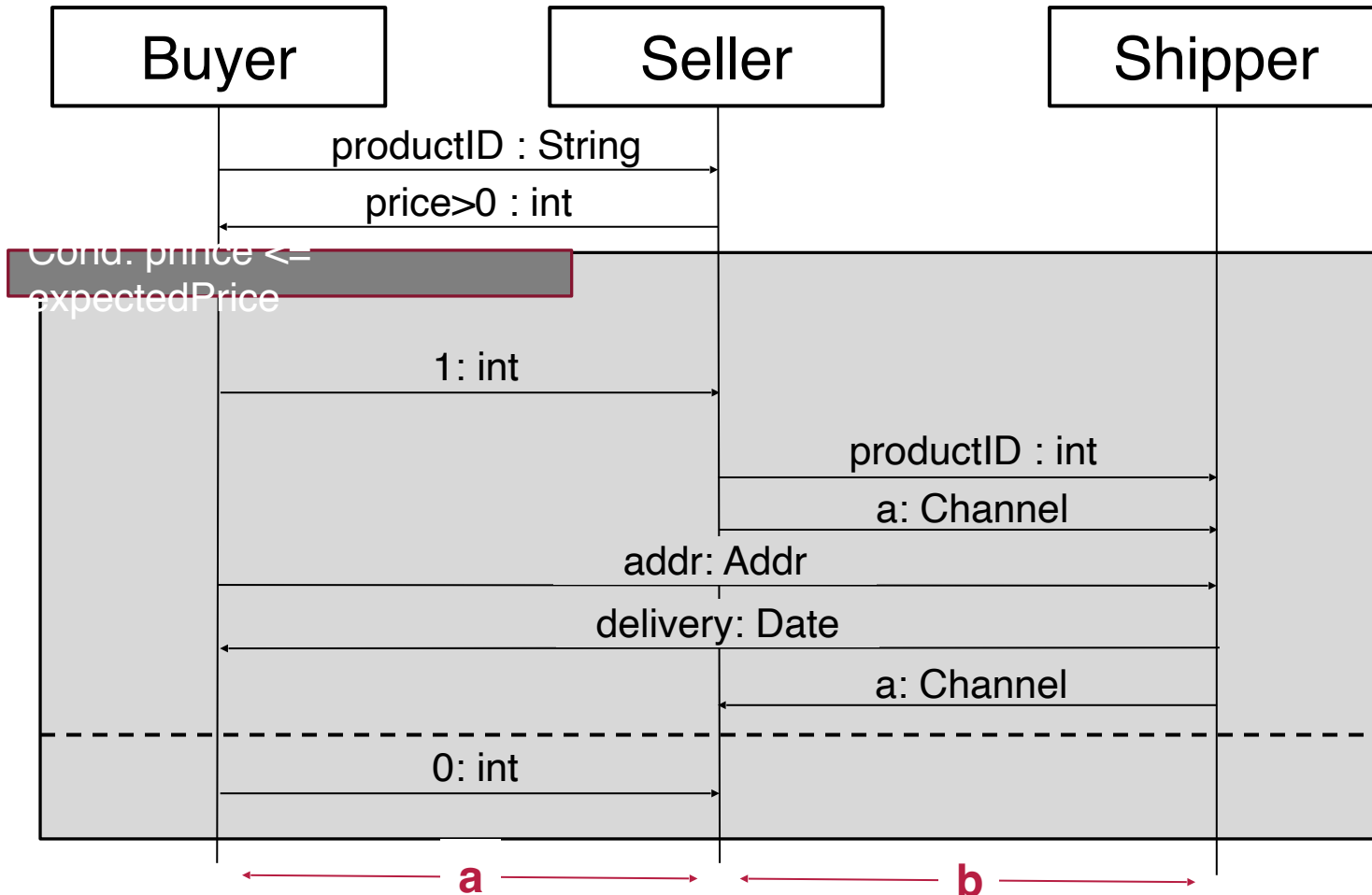


AUTOMATED VERIFICATION FOR RACE-FREE CHANNELS WITH IMPLICIT AND EXPLICIT SYNCHRONIZATION

Andreea Costea, Wei-Ngan Chin, Florin Craciun, Shengchao Qin

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CONTEXT – COMMUNICATION PROTOCOLS



Possible problems:

- deadlock
- unexpected communication
- transmission race

STATE OF THE ART

Behavioural types:

- Generic types¹: types and type environments as abstract processes, and then guarantee deadlock-freedom of process by checking the corresponding type environment.
- Behavioral separation²: extends separation logics and substructural types to higher order imperative concurrent programs in order to discipline interference
- Session types^{3,4}: Global and local types to describe communication and ensure deadlock freedom and race-freedom in the context of message passing

¹ IGARASHI , A. and KOBAYASHI , N., “A Generic Type System for the Pi-Calculus,” Theoretical Computer Science, vol. 311, no. 1, pp. 121 – 163, 2004.

² CAIRES , L. and SECO , J. C., “The Type Discipline of Behavioral Separation,” in POPL 2013.

³ HONDA , K., VASCONCELOS , V. T., and KUBO , M., “Language primitives and type discipline for structured communication-based programming,” in ESOP '98.

⁴ HONDA , K., YOSHIDA , N., and CARBONE , M., “Multiparty Asynchronous Session Types,” POPL 2008.

STATE OF THE ART (CONT.)

Logics with channel primitives:

- CSL for copyless message passing: an extension of separation for bidirectional communication between two players using global contracts
- CSL for pipelined parallelization⁶: an extension of separation logic which supports multiple players communicating through a single shared channel
- Chalice⁸ with support for message passing⁷: modular verification to prevent deadlocks of programs which mix message passing and locking.

⁵ VILLARD , J., LOZES , É., and CALCAGNO , C., “Proving copyless message passing,” in APLAS 2009 , pp. 194–209, Springer.

⁶ BELL , C. J., APPEL , A. W., and WALKER , D., “Concurrent Separation Logic for Pipelined Parallelization,” in SAS 2010, pp. 151–166, Springer.

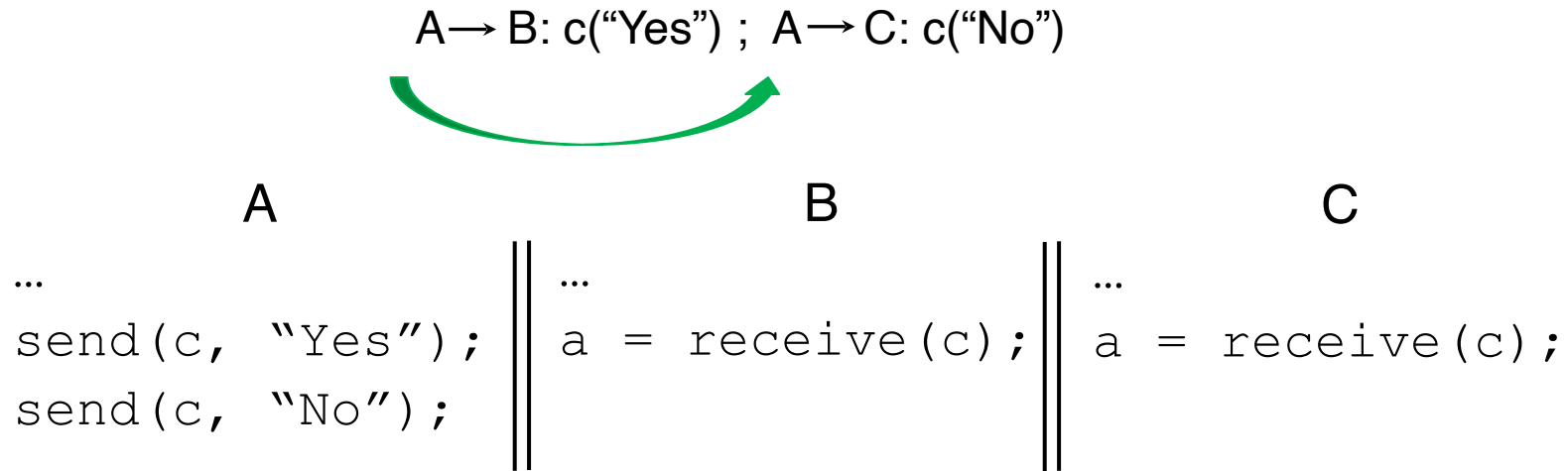
⁷ LEINO , K. R. M., MÜLLER , P., and SMANS , J., “Deadlock-Free Channels and Locks,” in ESOP 2010, pp. 407–426, Springer.

⁸ LEINO , K. R. M. and MÜLLER , P., “A Basis for Verifying Multi-Threaded Programs,” in ESOP 2009 pp. 378–393, Springer.

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AUTOMATED VERIFICATION FOR
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EXAMPLE 1



Communication assumptions:
shared FIFO message queues
unbounded queue
asynch communication

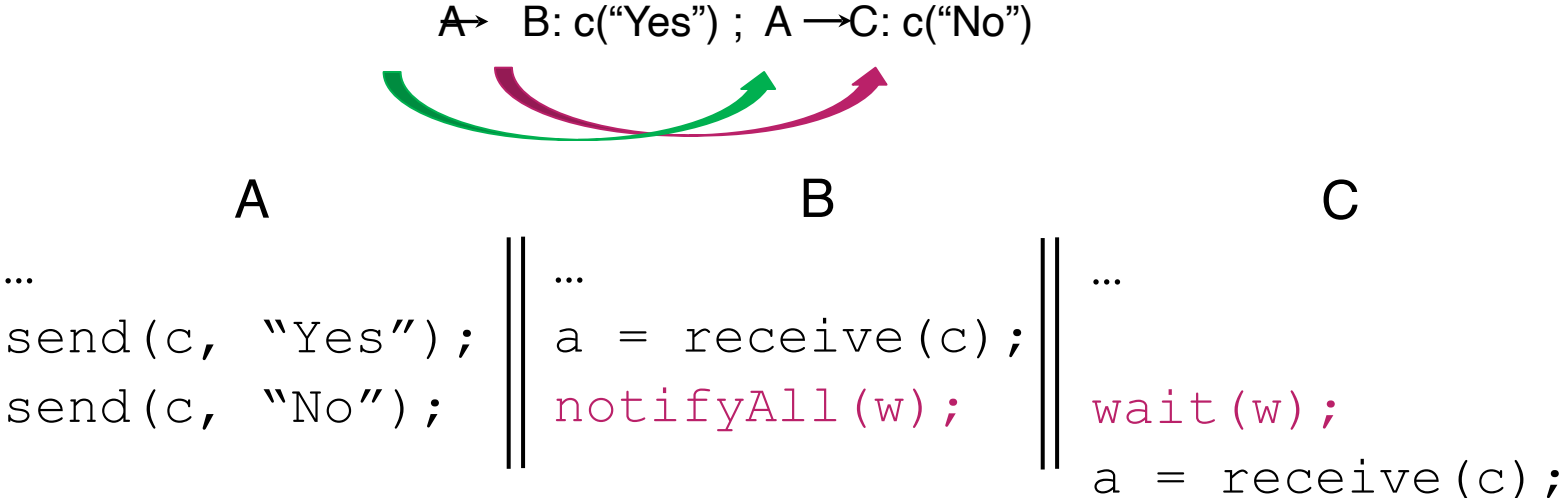
Who reads "Yes"?

Race on reading from c!

Current approaches declare this protocol as
UNSAFE

AUTOMATED VERIFICATION FOR
RACE-FREE CHANNELS WITH
IMPLICIT AND EXPLICIT
SYNCHRONIZATION

EXAMPLE 1



Communication assumptions:
shared FIFO message queues
 unbounded queue
asynch communication

Introduce a proof obligation on event ordering to prove that

B *happens-before* C

AUTOMATED VERIFICATION FOR
RACE-FREE CHANNELS WITH
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SYNCHRONIZATION

EXAMPLE 2

A \neg B: c("Yes") ; C \rightarrow B: c("No")



Race on writing to
c!

Introduce a proof obligation on event ordering to prove
that

A *happens-before* C

GOAL

$S_1 \quad R_1: c(\dots) ; \dots ; S_2 \quad R_2: c(\dots)$



To ensure race-freedom on c , prove that:

S_1 happens-before S_2

and

R_1 happens-before R_2

MERCURIUS: A LOGIC FOR PROTOCOL SPECIFICATION

<i>Single transmission</i>	$T ::= S \xrightarrow{i} R : c \langle v \cdot \Delta \rangle$
<i>Global protocol</i>	$G ::= T$
<i>Concurrency</i>	$G * G$
<i>Choice</i>	$G \vee G$
<i>Sequencing</i>	$G ; G$
<i>Guard</i>	$\ominus(\Psi)$
<i>Assumption</i>	$\oplus(\Psi)$
<i>Inaction</i>	emp

(Parties) $P, S, R \in \text{Role}$ (Channels) $c \in \text{Chan}$ (Messages) $v \cdot \Delta$ (Labels) $i \in \text{Nat}$

WELL-FORMEDNESS (*)

[Well-Formed Concurrency] A protocol specification, $G_1 * G_2$, is said to be well-formed with respect to $*$ if and only if $\forall c \in G_1 \implies c \notin G_2$, and vice versa.

WELL-FORMEDNESS (\forall)

(a) (same first channel) $\forall c_1 \in i_k, c_2 \in l_j \Rightarrow c_1 = c_2$;

(b) (same first sender S) $\forall S_1 \in i_k, S_2 \in l_j \Rightarrow S_1 = S_2 \wedge S = S_1$;

(c) (same first receiver R) $\forall R_1 \in i_k, R_2 \in l_j \Rightarrow R_1 = R_2 \wedge R = R_1$;

(d) (mutually exclusive "first" messages)

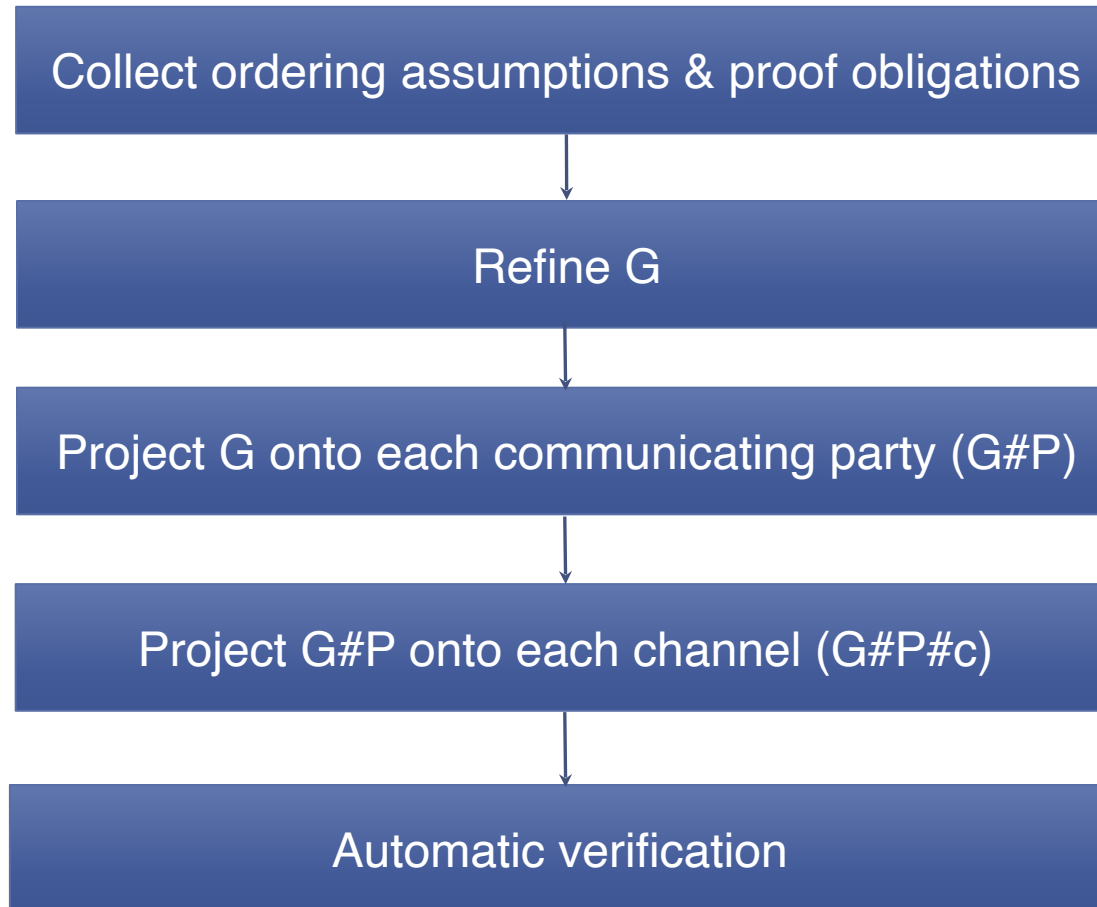
$$\forall j, k \in \{i_1, \dots, i_n, l_1, \dots, l_m\} \Rightarrow \text{UNSAT}(\Delta_j \wedge \Delta_k) \vee j = k;$$

(e) (same roles) $\forall P \in G_1 \vee G_2 \Rightarrow P = S \vee P = R$, with peers S and R the roles referenced by conditions (b) and (c), respectively;

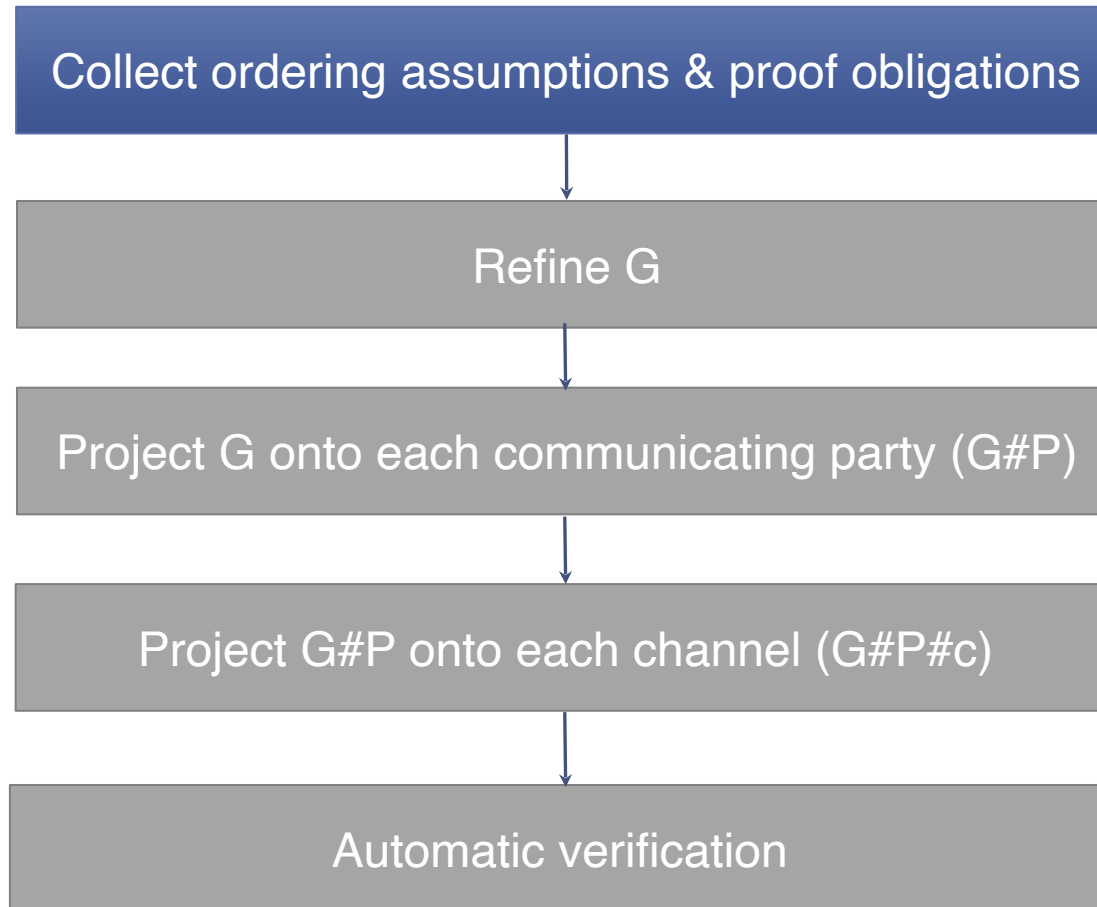
(f) (recursive well-formedness) G_1 and G_2 are well-formed with respect to \forall .

OVERVIEW OF OUR APPROACH

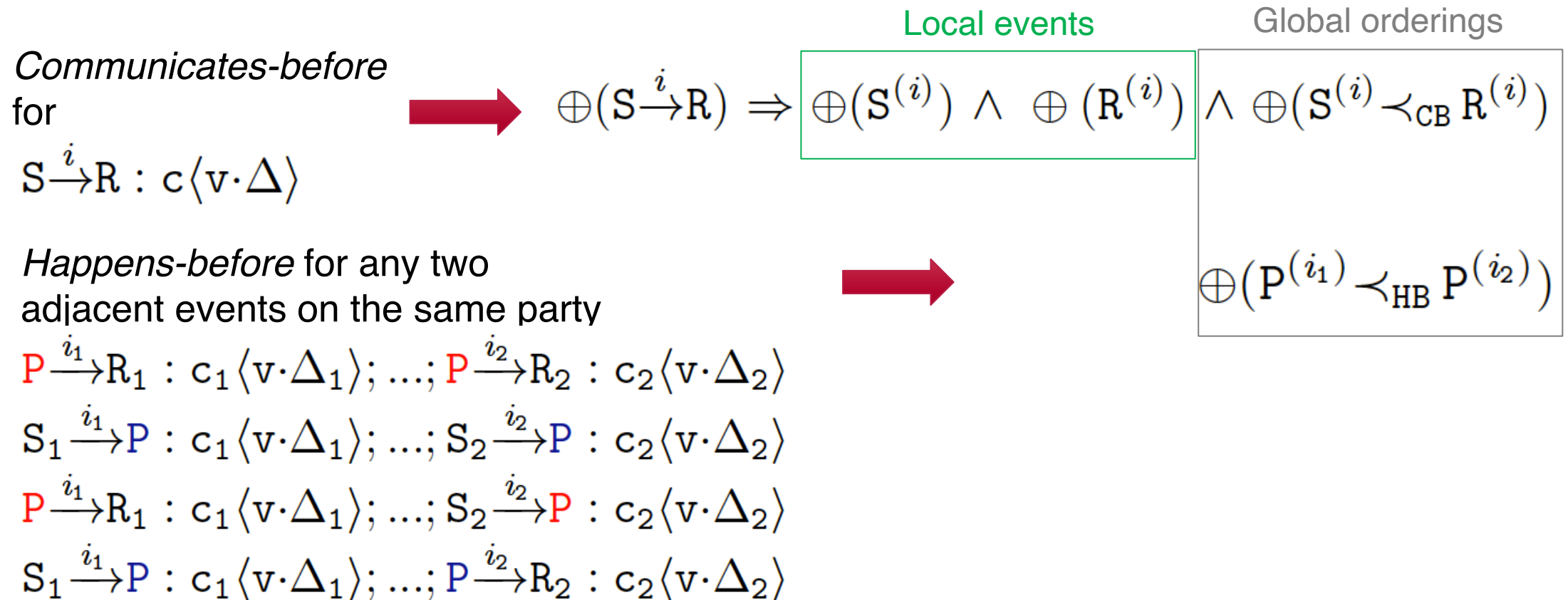
Given G:



OVERVIEW OF OUR APPROACH



ORDERING ASSUMPTIONS



RACE-FREE ASSERTIONS

$$S_1 \xrightarrow{i_1} R_1 : \mathbf{c} \langle v \cdot \Delta_1 \rangle; \dots; S_2 \xrightarrow{i_2} R_2 : \mathbf{c} \langle v \cdot \Delta_2 \rangle$$

Proof-obligation to check race-freedom of \mathbf{c} :

$$\ominus (S_1^{(i_1)} \prec_{\text{HB}} S_2^{(i_2)} \wedge R_1^{(i_1)} \prec_{\text{HB}} R_2^{(i_2)})$$

ORDERINGS CONSTRAINT SYSTEM

Send/Recv Event $E ::= P^{(i)}$
Ordering Constraints $\vartheta ::= E \prec_{\text{CB}} E \mid E \prec_{\text{HB}} E$
Race – Free Assertions $\Psi ::= E \mid \text{not}(E) \mid \vartheta \mid \Psi \wedge \Psi \mid \Psi \vee \Psi \mid E \Rightarrow \Psi$

$\Pi \models P^{(i)}$ iff $P^{(i)} \in \Pi$
 $\Pi \models \text{not}(P^{(i)})$ iff $P^{(i)} \notin \Pi$
 $\Pi \models P_1^{(i_1)} \prec_{\text{HB}} P_2^{(i_2)}$ iff $(\bigwedge_{\Psi_j \in \Pi} \Psi_j) \Rightarrow^* P_1^{(i_1)} \prec_{\text{HB}} P_2^{(i_2)}$
 $\Pi \models \Psi_1 \wedge \Psi_2$ iff $\Pi \models \Psi_1$ and $\Pi \models \Psi_2$
 $\Pi \models \Psi_1 \vee \Psi_2$ iff $\Pi \models \Psi_1$ or $\Pi \models \Psi_2$
 $\Pi \models E \Rightarrow \Psi$ iff $\Pi \models E \Rightarrow \Pi \models \Psi$

Constraint propagation lemmas:

$$P_1^{(i_1)} \prec_{\text{HB}} P_2^{(i_2)} \wedge P_2^{(i_2)} \prec_{\text{HB}} P_3^{(i_3)} \Rightarrow P_1^{(i_1)} \prec_{\text{HB}} P_3^{(i_3)} \quad (\text{HB-HB})$$

$$P_1^{(i_1)} \prec_{\text{CB}} P_2^{(i_1)} \wedge P_2^{(i_1)} \prec_{\text{HB}} P_3^{(i_2)} \Rightarrow P_1^{(i_1)} \prec_{\text{HB}} P_3^{(i_2)} \quad (\text{CB-HB})$$

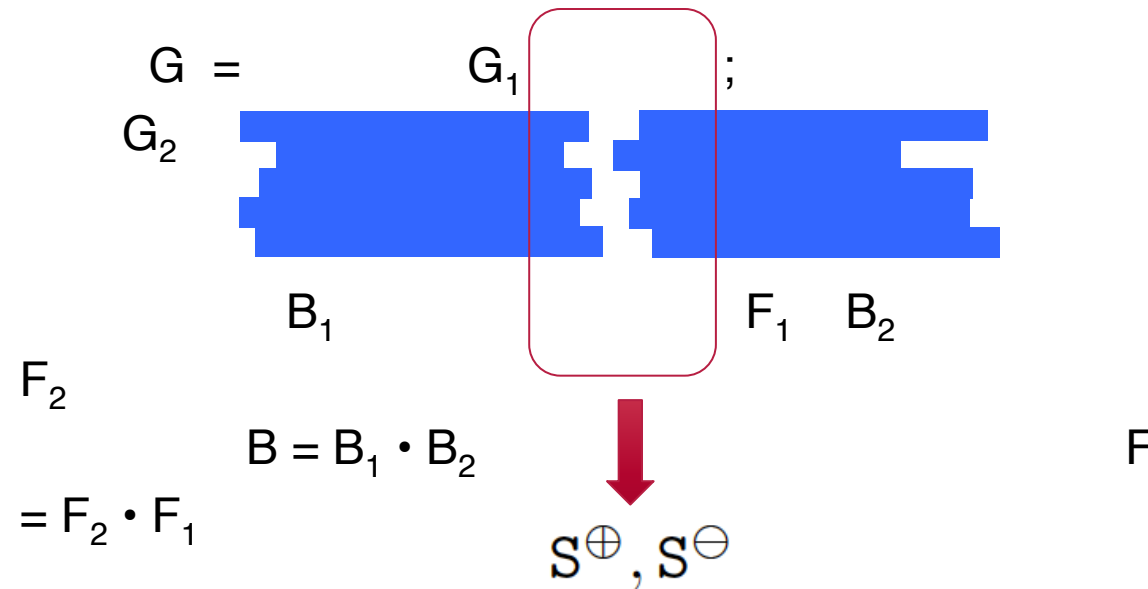
COLLECTION – BUILDING AND MERGING SUMMARIES

Summary := $B^{\text{border}} \times F^{\text{border}}$

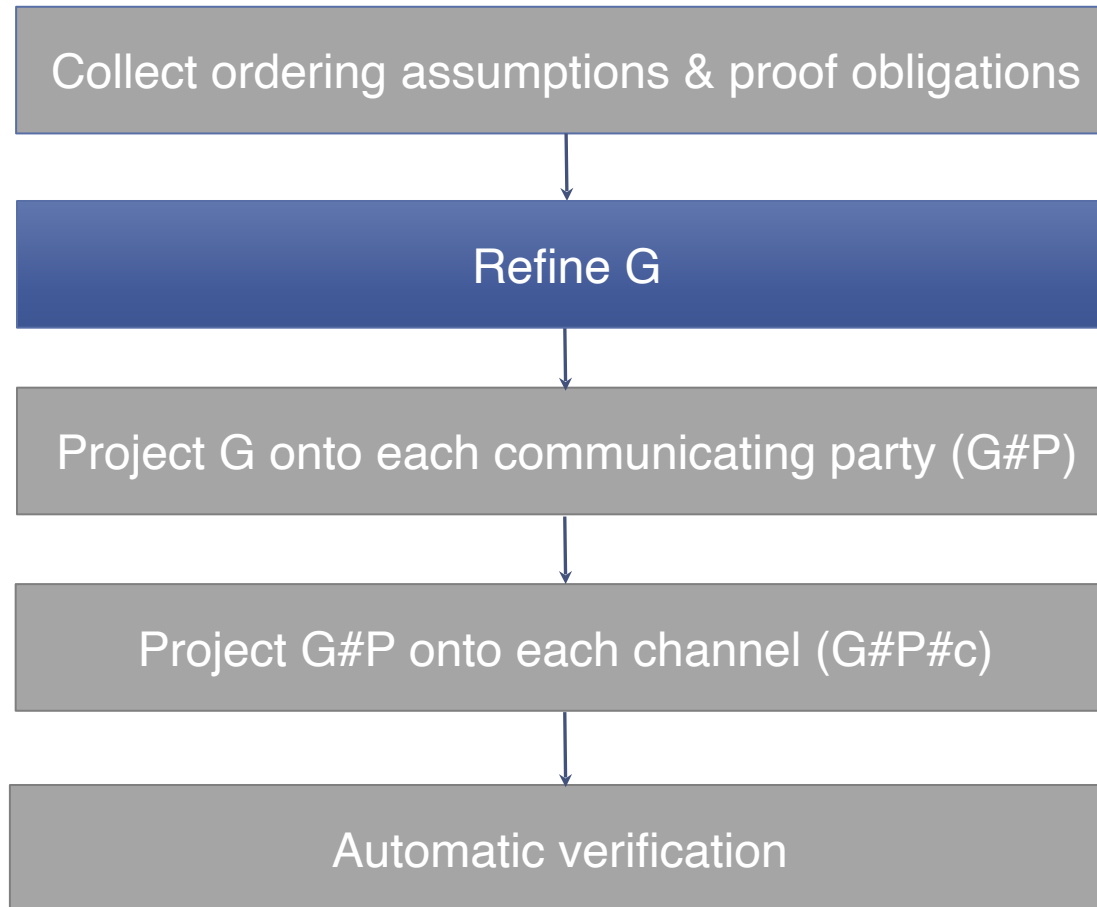
Border := $M^{\text{events}} \times M^{\text{trans}}$

M^{events} := $\text{Role} \rightarrow \text{Events}$

M^{trans} := $\text{Chan} \rightarrow \text{Trans}$



OVERVIEW OF OUR APPROACH



EXAMPLE 3

$$(A \xrightarrow{1} C : c \langle t_1 \rangle); (A \xrightarrow{2} B : c_2 \langle t_2 \rangle); (B \xrightarrow{3} C : c \langle t_3 \rangle)$$

The diagram shows three components: $(A \xrightarrow{1} C : c \langle t_1 \rangle)$, $(A \xrightarrow{2} B : c_2 \langle t_2 \rangle)$, and $(B \xrightarrow{3} C : c \langle t_3 \rangle)$. Green arrows point from the first component to the second, and from the second to the third. A purple arrow points from the first component to the third.

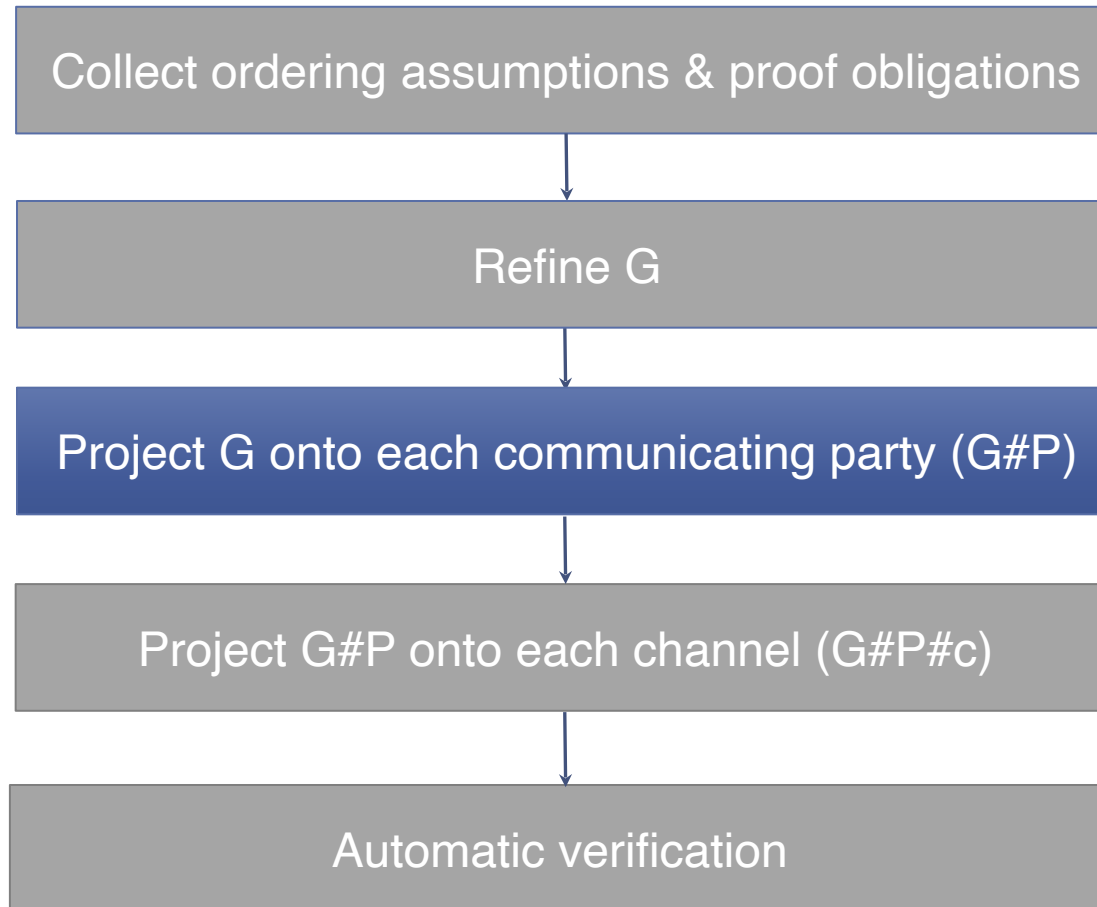
Spec

(i) Refinement

$A \xrightarrow{1} C : c \langle t_1 \rangle;$
 $A \xrightarrow{2} B : c_2 \langle t_2 \rangle;$
 $B \xrightarrow{3} C : c \langle t_3 \rangle$

$A \xrightarrow{1} C : c \langle t_1 \rangle; \oplus(A^{(1)}); \oplus(C^{(1)}); \oplus(A^{(1)} \prec_{CB} C^{(1)});$
 $A \xrightarrow{2} B : c_2 \langle t_2 \rangle; \oplus(A^{(2)}); \oplus(B^{(2)}); \oplus(A^{(1)} \prec_{HB} A^{(2)}); \oplus(A^{(2)} \prec_{CB} B^{(2)});$
 $B \xrightarrow{3} C : c \langle t_3 \rangle; \oplus(B^{(3)}); \oplus(C^{(3)}); \oplus(B^{(2)} \prec_{HB} B^{(3)}); \oplus(C^{(1)} \prec_{HB} C^{(3)}) \oplus (B^{(3)} \prec_{HB} C^{(3)}); \ominus(1 \prec_{HB} 3)$

OVERVIEW OF OUR APPROACH



GLOBAL SPEC \rightarrow PER PARTY (LANGUAGE)

Local protocol
Send/Receive
HO variable
Concurrency
Choice
Sequence
Guard/Assumption

$L^P ::=$
 $c!v \cdot \Delta \mid c?v \cdot \Delta$
 $\mid V$
 $\mid L^P * L^P$
 $\mid L^P \vee L^P$
 $\mid L^P ; L^P$
 $\mid \ominus(\Delta) \mid \oplus(\Delta)$

GLOBAL SPEC \rightarrow PER PARTY (PROJECTION RULES)

$$\begin{aligned}
 (P_1 \xrightarrow{i} P_2 : c \langle \Delta \rangle) \downarrow_P &:= \begin{cases} c!v \cdot \Delta & \text{if } P=P_1 \\ c?v \cdot \Delta & \text{if } P=P_2 \\ \text{emp} & \text{otherwise} \end{cases} & \begin{aligned} (G_1 * G_2) \downarrow_P &:= (G_1) \downarrow_P * (G_2) \downarrow_P \\ (G_1 \vee G_2) \downarrow_P &:= (G_1) \downarrow_P \vee (G_2) \downarrow_P \\ (G_1 ; G_2) \downarrow_P &:= (G_1) \downarrow_P ; (G_2) \downarrow_P \end{aligned} \\
 (\oplus(P_1^{(i)})) \downarrow_P &:= \begin{cases} \oplus(P^{(i)}) & \text{if } P=P_1 \\ \text{emp} & \text{otherwise} \end{cases} \\
 (\ominus(P_1^{(i_1)} \prec_{\text{HB}} P_2^{(i_2)})) \downarrow_P &:= \begin{cases} \ominus(P_1^{(i_1)} \prec_{\text{HB}} P_2^{(i_2)}) & \text{if } P=P_2 \\ \oplus(P_1^{(i_1)} \prec_{\text{HB}} P_2^{(i_2)}) & \text{otherwise} \end{cases}
 \end{aligned}$$

EXAMPLE 3: PER PARTY SPEC

$(A \xrightarrow{1} C : c \langle t_1 \rangle); (A \xrightarrow{2} B : c_2 \langle t_2 \rangle); (B \xrightarrow{3} C : c \langle t_3 \rangle)$

Spec

$A \xrightarrow{1} C : c \langle t_1 \rangle;$
 $A \xrightarrow{2} B : c_2 \langle t_2 \rangle;$
 $B \xrightarrow{3} C : c \langle t_3 \rangle$

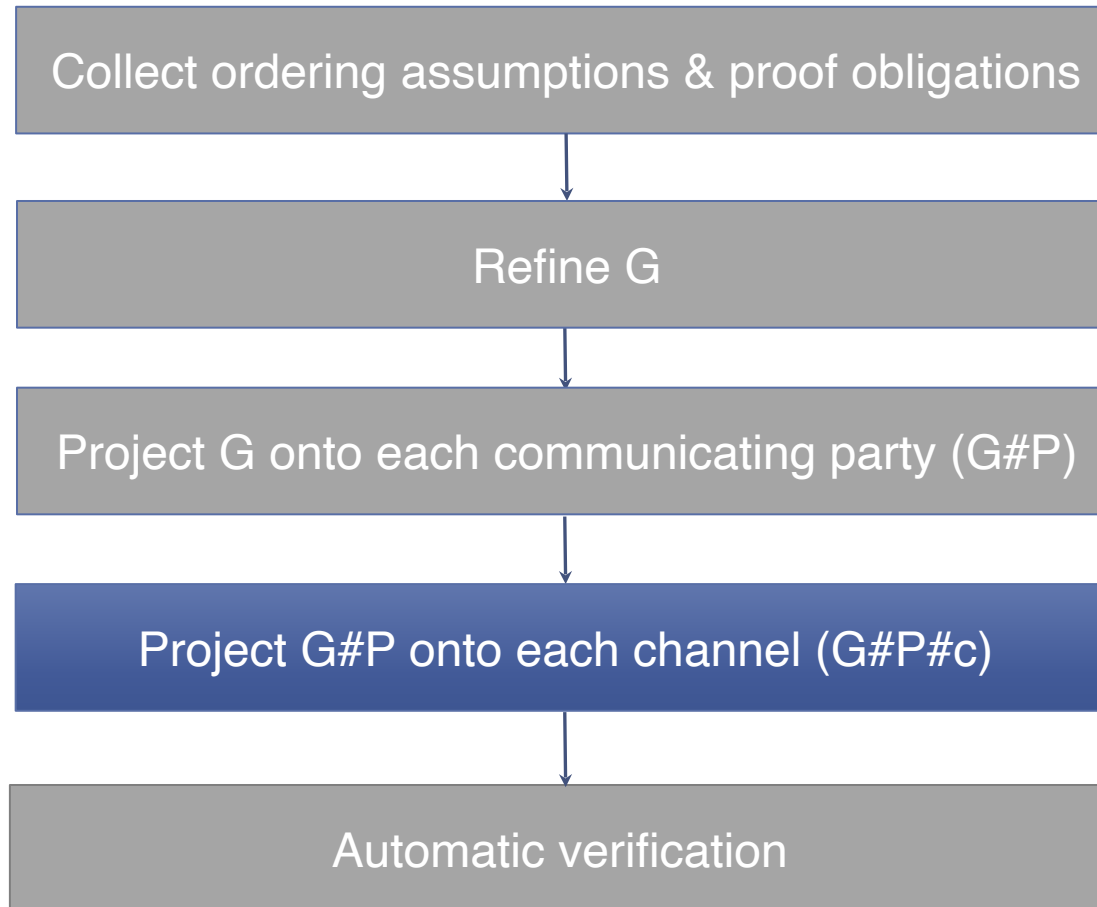
(i) Refinement

$A \xrightarrow{1} C : c \langle t_1 \rangle; \oplus(A^{(1)}); \oplus(C^{(1)}); \oplus(A^{(1)} \prec_{CB} C^{(1)});$
 $A \xrightarrow{2} B : c_2 \langle t_2 \rangle; \oplus(A^{(2)}); \oplus(B^{(2)}); \oplus(A^{(1)} \prec_{HB} A^{(2)}); \oplus(A^{(2)} \prec_{CB} B^{(2)});$
 $B \xrightarrow{3} C : c \langle t_3 \rangle; \oplus(B^{(3)}); \oplus(C^{(3)}); \oplus(B^{(2)} \prec_{HB} B^{(3)}); \oplus(C^{(1)} \prec_{HB} C^{(3)}) \oplus (B^{(3)} \prec_{HB} C^{(3)}); \ominus(1 \prec_{HB} 3)$

(ii) Per-Party Projection

$G\#A \triangleq c!t_1; \oplus(A^{(1)}); c_2!t_2; \oplus(A^{(2)}); \ominus(1 \prec_{HB} 3)_A$	$G\#All \triangleq \oplus(A^{(1)} \prec_{CB} C^{(1)}); \oplus(A^{(1)} \prec_{HB} A^{(2)});$
$G\#B \triangleq c_2?t_2; \oplus(B^{(2)}); c!t_3; \oplus(B^{(3)}); \ominus(1 \prec_{HB} 3)_B$	$\oplus(A^{(2)} \prec_{CB} B^{(2)}); \oplus(B^{(2)} \prec_{HB} B^{(3)});$
$G\#C \triangleq c?t_1; \oplus(C^{(1)}); c?t_3; \oplus(C^{(3)}); \ominus(1 \prec_{HB} 3)_C$	$\oplus(C^{(1)} \prec_{HB} C^{(3)}); \oplus(B^{(3)} \prec_{CB} C^{(3)})$

OVERVIEW OF OUR APPROACH



PER PARTY \rightarrow PER CHANNEL (LANGUAGE)

<i>Local protocol</i>	$L ::=$
<i>Send/Receive</i>	$!v \cdot \Delta \mid ?v \cdot \Delta$
<i>HO variable</i>	$\mid V$
<i>Choice</i>	$\mid L \vee L$
<i>Sequence</i>	$\mid L; L$
<i>Guard/Assumption</i>	$\mid \ominus(\Delta) \mid \oplus(\Delta)$

PER PARTY \rightarrow PER CHANNEL (PROJECTION RULES)

$$\begin{aligned}
 (c_1!v \cdot \Delta)|_c &:= \begin{cases} !v \cdot \Delta & \text{if } c=c_1 \\ \text{emp} & \text{otherwise} \end{cases} & (L_1^P * L_2^P)|_c &:= \begin{cases} (L_j^P)|_c & \text{if } c \in L_j, j=1 \text{ or } 2 \\ \text{emp} & \text{otherwise} \end{cases} \\
 (c_1?v \cdot \Delta)|_c &:= \begin{cases} ?v \cdot \Delta & \text{if } c=c_1 \\ \text{emp} & \text{otherwise} \end{cases} & (L_1^P \vee L_2^P)|_c &:= (L_1^P)|_c \vee (L_2^P)|_c \\
 & & (L_1^P; L_2^P)|_c &:= (L_1^P)|_c; (L_2^P)|_c
 \end{aligned}$$

$$\begin{aligned}
 (\oplus(P^{(i)}))|_c &:= \begin{cases} \oplus(P^{(i)}) & \text{if } c \in i \\ \ominus(P^{(i)}) & \text{otherwise} \end{cases} \\
 (\ominus(P_1^{(i_1)} \prec_{\text{HB}} P_2^{(i_2)}))|_c &:= \begin{cases} \ominus(P_1^{(i_1)} \prec_{\text{HB}} P_2^{(i_2)}) & \text{if } c \in i_2 \\ \text{emp} & \text{otherwise} \end{cases} \\
 (\oplus(P_1^{(i_1)} \prec_{\text{HB}} P_2^{(i_2)}))|_c &:= \begin{cases} \oplus(P_1^{(i_1)} \prec_{\text{HB}} P_2^{(i_2)}) & \text{if } c \in i_2 \\ \text{emp} & \text{otherwise} \end{cases}
 \end{aligned}$$

EXAMPLE 3: PER CHANNEL SPEC

$$(A \xrightarrow{1} C : c \langle t_1 \rangle); (A \xrightarrow{2} B : c_2 \langle t_2 \rangle); (B \xrightarrow{3} C : c \langle t_3 \rangle)$$

Spec

$$\begin{array}{l} A \xrightarrow{1} C : c \langle t_1 \rangle; \\ A \xrightarrow{2} B : c_2 \langle t_2 \rangle; \\ B \xrightarrow{3} C : c \langle t_3 \rangle \end{array}$$

(i) Refinement

$$\begin{array}{l} A \xrightarrow{1} C : c \langle t_1 \rangle; \oplus(A^{(1)}); \oplus(C^{(1)}); \oplus(A^{(1)} \prec_{CB} C^{(1)}); \\ A \xrightarrow{2} B : c_2 \langle t_2 \rangle; \oplus(A^{(2)}); \oplus(B^{(2)}); \oplus(A^{(1)} \prec_{HB} A^{(2)}); \oplus(A^{(2)} \prec_{CB} B^{(2)}); \\ B \xrightarrow{3} C : c \langle t_3 \rangle; \oplus(B^{(3)}); \oplus(C^{(3)}); \oplus(B^{(2)} \prec_{HB} B^{(3)}); \oplus(C^{(1)} \prec_{HB} C^{(3)}) \oplus (B^{(3)} \prec_{HB} C^{(3)}); \ominus(1 \prec_{HB} 3) \end{array}$$

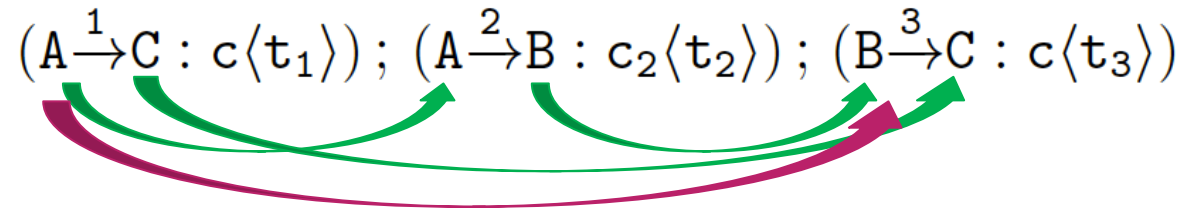
(ii) Per-Party Projection

$G\#A \triangleq c!t_1; \oplus(A^{(1)}); c_2!t_2; \oplus(A^{(2)}); \ominus(1 \prec_{HB} 3)_A$	$G\#All \triangleq \oplus(A^{(1)} \prec_{CB} C^{(1)}); \oplus(A^{(1)} \prec_{HB} A^{(2)});$
$G\#B \triangleq c_2?t_2; \oplus(B^{(2)}); c!t_3; \oplus(B^{(3)}); \ominus(1 \prec_{HB} 3)_B$	$\oplus(A^{(2)} \prec_{CB} B^{(2)}); \oplus(B^{(2)} \prec_{HB} B^{(3)});$
$G\#C \triangleq c?t_1; \oplus(C^{(1)}); c?t_3; \oplus(C^{(3)}); \ominus(1 \prec_{HB} 3)_C$	$\oplus(C^{(1)} \prec_{HB} C^{(3)}); \oplus(B^{(3)} \prec_{CB} C^{(3)})$

(ii) Per-Channel Projection

$G\#A\#c \triangleq !t_1; \oplus(A^{(1)}); \oplus(A^{(1)} \prec_{HB} B^{(3)}); \oplus(C^{(1)} \prec_{HB} C^{(3)})$	$G\#A\#c_2 \triangleq \ominus(A^{(1)}); !t_2; \oplus(A^{(2)})$
$G\#B\#c \triangleq \ominus(B^{(2)}); !t_3; \oplus(B^{(3)}); \ominus(A^{(1)} \prec_{HB} B^{(3)}); \oplus(C^{(1)} \prec_{HB} C^{(3)})$	$G\#B\#c_2 \triangleq ?t_2; \oplus(B^{(2)})$
$G\#C\#c \triangleq ?t_1; \oplus(C^{(1)}); ?t_3; \oplus(C^{(3)}); \ominus(C^{(1)} \prec_{HB} C^{(3)}); \oplus(A^{(1)} \prec_{HB} B^{(3)})$	

EXAMPLE 3: PER CHANNEL SPEC

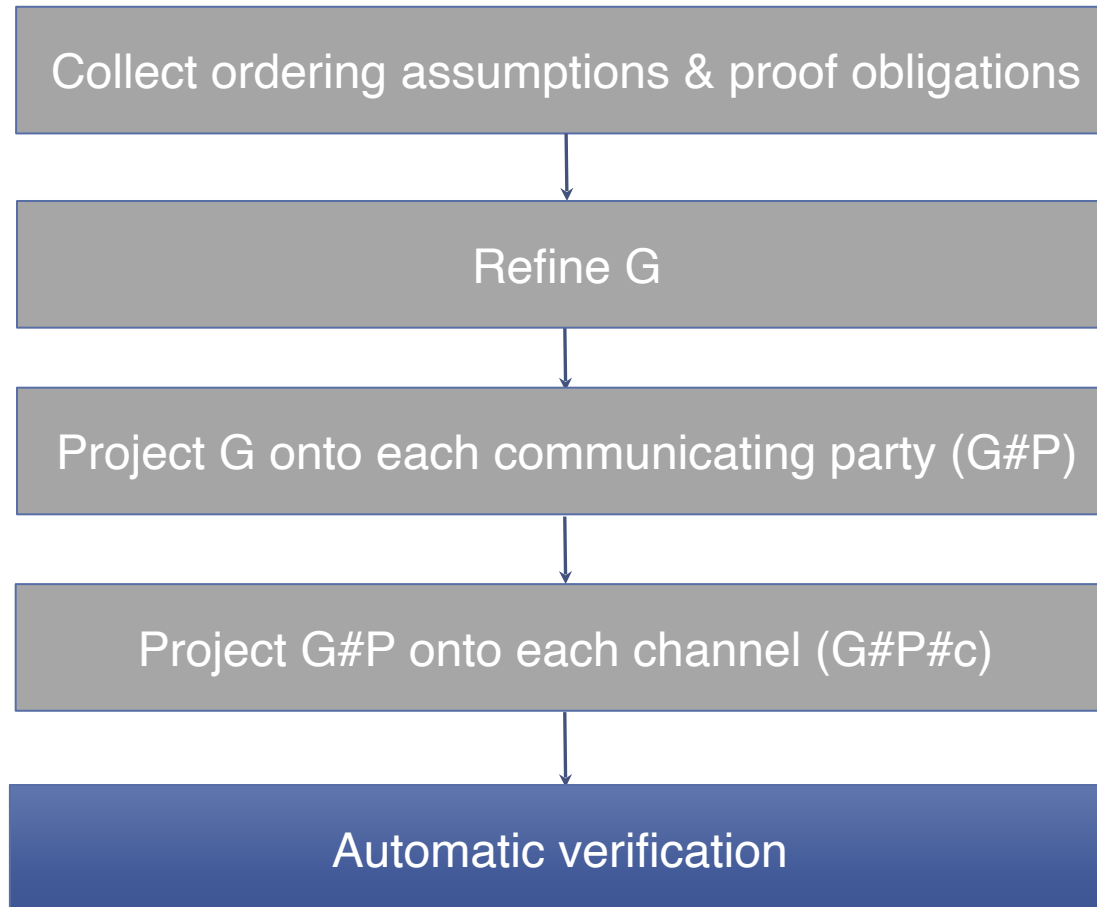


$$\ominus(1 \prec_{\text{HB}} 3)_{\text{B}} = \ominus(A^{(1)} \prec_{\text{HB}} B^{(3)}); \oplus(C^{(1)} \prec_{\text{HB}} C^{(3)})$$

$$\ominus(1 \prec_{\text{HB}} 3)_{\text{C}} = \ominus(C^{(1)} \prec_{\text{HB}} C^{(3)}); \oplus(A^{(1)} \prec_{\text{HB}} B^{(3)})$$

$$\ominus(1 \prec_{\text{HB}} 3)_{\text{A}} = \oplus(A^{(1)} \prec_{\text{HB}} B^{(3)}); \oplus(C^{(1)} \prec_{\text{HB}} C^{(3)})$$

OVERVIEW OF OUR APPROACH



COMMUNICATION PRIMITIVES

$$\frac{\text{[OPEN]}}{V_{\text{pre}} = \mathcal{C}(c_1, P_1, L_1) * \dots * \mathcal{C}(c_1, P_n, L_n) \quad V_{\text{post}} = \mathcal{C}(c, P_1, L_1) * \dots * \mathcal{C}(c, P_n, L_n)} \\ \{V_{\text{pre}}\} \mathbf{c = open()} \text{ with } (c_1, P_1..P_n) \{V_{\text{post}} \wedge \text{bind}(c, c_1)\}$$
$$\frac{\text{[CLOSE]}}{V_{\text{pre}} = \mathcal{C}(c, P_1, \text{emp}) * \dots * \mathcal{C}(c, P_n, \text{emp}) \quad V_{\text{post}} = \mathcal{C}(c_1, P_1, \text{emp}) * \dots * \mathcal{C}(c_1, P_n, \text{emp})} \\ \{V_{\text{pre}} \wedge \text{bind}(c, c_1)\} \mathbf{close}(c) \{V_{\text{post}}\}$$
$$\text{[SEND]} \\ \{\mathcal{C}(c, P, !v \cdot V(v); L) * V(x) \wedge \text{Peer}(P)\} \mathbf{send}(c, x) \{\mathcal{C}(c, P, L)\}$$
$$\text{[RECV]} \\ \{\mathcal{C}(c, P, ?v \cdot V(v); L) \wedge \text{Peer}(P)\} \mathbf{x = receive}(c) \{V(x) * \mathcal{C}(c, P, L)\}$$

EXAMPLE 3 - VERIFICATION

$$\begin{aligned} & \{ \text{Common}(G\#All) * \text{Party}(A, G\#A) * \text{Party}(B, G\#B) * \text{Party}(C, G\#C) \} \\ & \quad (\text{Code}_A \parallel \text{Code}_B \parallel \text{Code}_C) \\ & \quad \{ \text{Party}(A, \text{emp}) * \text{Party}(B, \text{emp}) * \text{Party}(C, \text{emp}) \} \end{aligned}$$

“Release” lemma:

$$\text{Party}(B, G\#B) \Leftrightarrow \mathcal{C}(c, B, G\#B\#s) * \mathcal{C}(c_1, B, G\#B\#c_1)$$

“Join-emp” lemma:

$$\text{Party}(B, \text{emp}) \Leftrightarrow \mathcal{C}(c, B, \text{emp}) * \mathcal{C}(c_1, B, \text{emp})$$

FINAL REMARKS

Race-freedom via implicit & explicit synchronization

Ordering constraint system

Expressive session logic, which goes beyond types

More in the technical report

- Well-formedness of $*$ and \vee
- Explicit synchronization specifications
- Recursion
- Full constraint system
- Entailment rules

WAIT-NOTIFYALL PRIMITIVES

$$\frac{\begin{array}{c} \text{[CREATE]} \\ V = \bigwedge_{j \in \{2..n\}} \oplus (E_j \Rightarrow E_1 \prec_{\text{HB}} E_j) \end{array}}{\{\text{emp}\} \mathbf{w} = \text{create}() \text{ with } \mathbf{E}_1, \overline{\mathbf{E}_2.. \mathbf{E}_n} \{ \text{NOTIFY}(\mathbf{w}, \ominus(\mathbf{E}_1)) * \text{WAIT}(\mathbf{w}, V) \}}$$

$$\frac{\text{[NOTIFY-ALL]} \{ \text{NOTIFY}(\mathbf{w}, \ominus(\mathbf{E}_1)) \wedge \mathbf{E}_1 \} \mathbf{notifyAll}(\mathbf{w}) \{ \text{NOTIFY}(\mathbf{w}, \text{emp}) \}}{\text{[WAIT]}}$$

$$\frac{V^{\text{rel}} = \oplus (E_2 \Rightarrow E_1 \prec_{\text{HB}} E_2)}{\{ \text{WAIT}(\mathbf{w}, V^{\text{rel}}) \wedge \text{not}(E_2) \} \mathbf{wait}(\mathbf{w}) \{ \text{WAIT}(\mathbf{w}, \text{emp}) * V^{\text{rel}} \}}$$

$$(\text{Wait lemma}) \quad \oplus (E_2 \Rightarrow E_1 \prec_{\text{HB}} E_2) \wedge E_2 \Rightarrow E_1 \prec_{\text{HB}} E_2$$

$$(\text{Distribute-waits lemma}) \quad \text{WAIT}(\mathbf{w}, \bigwedge_{j \in \{2..n\}} \Psi_j) \Rightarrow \bigwedge_{j \in \{2..n\}} \text{WAIT}(\mathbf{w}, \Psi_j)$$

Deadlock-check:

$$\text{NOTIFY}(\mathbf{w}, \ominus(\mathbf{E}_1)) * \text{WAIT}(\mathbf{w}, \text{emp}) \Rightarrow \text{false}$$

MERCURIUS: SPECIFICATION LANGUAGE

<i>Symbolic pred.</i>	$pred$	$::= p(\text{root}, v^*) \equiv \Phi \mid p(P^*, v^*) \equiv G$
<i>Formula</i>	Φ	$::= \bigvee \Delta \quad \Delta ::= \exists v^* \cdot \kappa \wedge \pi \mid \Delta * \Delta$
<i>Separation</i>	κ	$::= \text{emp} \mid v \mapsto d(v^*) \mid p(v^*) \mid C(v, P, L) \mid \kappa * \kappa \mid V$
<i>Pure</i>	π	$::= v : t \mid b \mid a \mid \phi \wedge \phi \mid \phi \vee \phi \mid \neg \phi$ $\mid \exists v \cdot \phi \mid \forall v \cdot \phi \mid \gamma$
<i>Pointer eq./diseq.</i>	γ	$::= v = v \mid v = \text{null} \mid v \neq v \mid v \neq \text{null}$
<i>Boolean</i>	b	$::= \text{true} \mid \text{false} \mid b = b \quad a ::= s = s \mid s \leq s \mid V = \Delta$
<i>Presburger Arith.</i>	s	$::= k^{\text{int}} \mid v \mid k^{\text{int}} \times s \mid s + s \mid -s$

where k^{int} : integer constant; v : first order variable;
 V : second-order variable; P : session role
 d : name of a user-defined data structure
 L : local protocol (defined in Fig. 5)